

EHU-96-6 Revised  
August 1996

# Is CP Violation Observable in Long Baseline Neutrino Oscillation Experiments ?

Morimitsu TANIMOTO <sup>1</sup>

*Science Education Laboratory, Ehime University, 790 Matsuyama, JAPAN*

## ABSTRACT

We have studied  $CP$  violation originated by the phase of the neutrino mixing matrix in the long baseline neutrino oscillation experiments. The direct measurements of  $CP$  violation is the difference of the transition probabilities between  $CP$ -conjugate channels. In those experiments, the  $CP$  violating effect is not suppressed if the highest neutrino mass scale is taken to be  $1 \sim 5\text{eV}$ , which is appropriate for the cosmological hot dark matter. Assuming the hierarchy for the neutrino masses, the upper bounds of  $CP$  violation have been calculated for three cases, in which mixings are constrained by the recent short baseline ones. The calculated upper bounds are larger than  $10^{-2}$ , which will be observable in the long baseline accelerator experiments. The matter effect, which is not  $CP$  invariant, has been also estimated in those experiments.

---

<sup>1</sup>E-mail address: [tanimoto@edserv.ed.ehime-u.ac.jp](mailto:tanimoto@edserv.ed.ehime-u.ac.jp)

# 1 Introduction

The origin of  $CP$  violation is still an open problem in particle physics. In the quark sector,  $CP$  violation has been intensively studied in the KM standard model [1]. For the lepton sector,  $CP$  violation is also expected unless the neutrinos are massless. In particular,  $CP$  violation in the neutrino flavor oscillations is an important phenomenon because it relates directly to the  $CP$  violating phase parameter in the mixing matrix for the massive neutrinos [2]. Unfortunately, this  $CP$  violating effect is suppressed in the short baseline accelerator experiments if the neutrinos have the hierarchical mass spectrum. However, the suppression is avoidable in the long baseline accelerator experiments, which are expected to operate in the near future [3] [4]. So one has a chance to observe the  $CP$  violating effect in those experiments.

The recent indications of a deficit in the  $\nu_\mu$  flux of the atmospheric neutrinos [5]-[7] has renewed interest in using accelerator neutrinos to perform the long baseline neutrino oscillation experiments. Many possibilities of experiments have been discussed [3]. The purpose of this paper is to present the numerical study of  $CP$  violation in those accelerator experiments.

There are only two hierarchical mass difference scales  $\Delta m^2$  in the three-flavor mixing scheme without introducing sterile neutrinos. If the highest neutrino mass scale is taken to be  $1 \sim 5\text{eV}$ , which is appropriate for the cosmological hot dark matter(HDM) [8], the other mass scale is either the atmospheric neutrino mass scale  $\Delta m^2 \simeq 10^{-2}\text{eV}^2$  [5]-[7] or the solar neutrino one  $\Delta m^2 \simeq 10^{-5} \sim 10^{-6}\text{eV}^2$  [9]. Since the long baseline experiments correspond to the atmospheric neutrino mass scale, we take  $\Delta m^2 \simeq 10^{-2}\text{eV}^2$  as the lower mass scale. The solar neutrino problem is not discussed in this paper. The solar one may be solved by introducing the sterile neutrino [10].

Our study of  $CP$  violation is presented in the framework of above pattern of the neutrino mass spectrum <sup>2</sup>. We also investigate the matter effect in the long baseline accelerator experiments since the background matter effect is not  $CP$  invariant. If the matter effect is not negligibly small compared to the  $CP$  violating effect in the vacuum, one should consider how to extract the matter effect from the data. It is found that the matter effect strongly depends on the hierarchical pattern of the neutrino masses and mixings.

## 2 CP Violation in Neutrino Flavor Oscillations

The amplitude of  $\nu_\alpha \rightarrow \nu_\beta$  transition with the neutrino energy  $E$  after traversing the distance  $L$  can be written as

$$\mathcal{A}(\nu_\alpha \rightarrow \nu_\beta) = e^{-iEL} \left\{ \delta_{\alpha\beta} + \sum_{k=2}^3 U_{\alpha k} U_{\beta k}^* \left[ \exp \left( -i \frac{\Delta m_{k1}^2 L}{2E} \right) - 1 \right] \right\} , \quad (1)$$

where  $\Delta m_{ij}^2 = m_i^2 - m_j^2$  is defined, and  $U_{\alpha i}$  denote the elements of the  $3 \times 3$  neutrino flavor mixing matrix, in which  $\alpha$  and  $i$  refer to the flavor eigenstate and the mass eigenstate, respectively. The amplitude  $\mathcal{A}(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$  is given by replacing  $U$  with  $U^*$  in the right hand side in eq.(1). The direct measurements of  $CP$  violation originated by the phase of the neutrino mixing matrix are the differences of the transition probabilities between  $CP$ -conjugate channels [2]:

$$\begin{aligned} \Delta P &\equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) - P(\nu_\mu \rightarrow \nu_e) = P(\nu_\mu \rightarrow \nu_\tau) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) \\ &= P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) - P(\nu_e \rightarrow \nu_\tau) = 4J_{CP}'(\sin D_{12} + \sin D_{23} + \sin D_{31}) , \end{aligned} \quad (2)$$

where

$$D_{ij} = \Delta m_{ij}^2 \frac{L}{2E} , \quad (3)$$

---

<sup>2</sup>  $CP$  and  $T$  violations have been studied in the case of  $\Delta m_{31}^2 \sim 10^{-2} \text{eV}^2$  [11].

and  $J_{CP}^\nu$  is defined for the rephasing invariant quantity of  $CP$  violation in the neutrino mixing matrix as well as the one in the quark sector [12]. In terms of the standard parametrization of the mixing matrix [13], we have

$$J_{CP}^\nu = \text{Im}(U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2}) = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \phi, \quad (4)$$

where  $\phi$  is the  $CP$  violating phase. The oscillatory terms are periodic in  $L/E$  and  $D_{12} + D_{23} + D_{31} = 0$  is satisfied.

For the neutrino masses, we expect the typical hierarchical relation  $\Delta m_{31}^2 \gg \Delta m_{32}^2$  or  $\Delta m_{31}^2 \gg \Delta m_{21}^2$  in order to guarantee two different mass scales. The former relation(hierarchy I) corresponds to  $m_3 \simeq m_2 \gg m_1$  and the latter one(hierarchy II) to  $m_3 \gg m_2 \gg m_1$ (or  $\simeq m_1$ ). The highest neutrino mass scale is taken to be  $m_3 = 1 \sim 5\text{eV}$ , which is appropriate for the cosmological HDM[8]. Those mass hierarchies have been discussed in the context of the solar neutrino problem, atmospheric neutrino one and HDM(as well as LSND [14]) by several authors [10][15]. Although their results seem to support hierarchy I, both cases of hierarchies I and II are studied in this paper.

Recently, there are significant short baseline accelerator experiments [14]-[17], in which the value of  $L/E$  is fixed for each one. For example,  $L = 30\text{m}$  and  $E = 36 \sim 60\text{MeV}$  are taken for the  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  experiment at LSND [14] and  $L = 800\text{m}$  and  $E = 30\text{GeV}$  for the  $\nu_\mu \rightarrow \nu_\tau$  experiment at CHORUS and MOMAD [17]. In these experiments, the value of  $D_{31}(\simeq -D_{12})$  is  $1 \sim 10$  with  $|D_{23}| \ll 1$  for hierarchy I ( $D_{31} \simeq -D_{23}$  is  $1 \sim 10$  and with  $|D_{12}| \ll 1$  for hierarchy II). Therefore the factor  $(\sin D_{12} + \sin D_{23} + \sin D_{31})$  is suppressed because two largest terms almost cancel due to opposite signs. Another term is still small. Then the  $CP$  violating effect in eq.(2) is significantly reduced due to this suppression. So one has no chance to observe  $CP$  violation for the present in the short baseline neutrino oscillation experiments.

However, the situation is very different in the long baseline accelerator experiments. Let us consider the case of hierarchy I for the neutrino masses. The oscillatory terms  $\sin D_{12}$  and  $\sin D_{31}$  can be replaced by the average value 0 since the magnitude of  $D_{31}(\simeq -D_{12})$  is  $10^3 \sim 10^4$ . Then  $CP$  violation is dominated by the  $\sin D_{23}$  term, which is not small because of  $|D_{23}| \simeq 1$ . The same situation is kept for hierarchy II. Thus the  $CP$  violating quantity  $\Delta P$  is not suppressed unless  $J_{CP}^\nu$  is very small.

### 3 Non-Suppression of $CP$ Violation in Long Baseline Accelerator Experiments

The long baseline accelerator experiments are planned to operate in the near future [3][4]. The most likely possibilities are in KEK-SuperKamiokande(250Km), CERN-Gran Sasso(730Km) and Fermilab-Soudan 2(730Km) experiments(MINOS). The sensitivity of the observable transition probability is expected to be  $10^{-2}$ . The average energy of the  $\nu_\mu$  beams are approximately 1GeV, 6GeV and 10GeV at KEK-PS(12GeV), CERN-SPS(80GeV) and Fermilab proton accelerator(120GeV), respectively. We can estimate  $S_{CP} \equiv \sin D_{12} + \sin D_{23} + \sin D_{31}$  in those experiments. The neutrino energy  $E$  dependences of  $S_{CP}$  are shown by solid curves for fixed  $L = 250\text{Km}$  in fig.(a) and for  $L = 730\text{Km}$  in fig.(b), where  $S_{CP}$  is averaged over energy spread of 20% at the reference energy. Here  $\Delta m_{31}^2 = 2.25\text{eV}^2$  and  $\Delta m_{32}^2(\Delta m_{21}^2) = 10^{-2}\text{eV}^2$  are taken in hierarchy I(II). Although our results depend on the value of  $\Delta m_{31}^2$ , these change only 5% for  $\Delta m_{31}^2 = 1 \sim 25\text{eV}^2$ . We also show the oscillation function  $\sin^2(\Delta m_{32}^2 L/4E)$ , which is important for the absolute value of the transition probability  $P(\nu_\alpha \rightarrow \nu_\beta)$  at those experiments, by dashed curves in fig.1.

**Fig. 1(a) and (b)**

As seen in fig.1(a), the absolute value of  $S_{CP}$  is almost maximum at  $E \simeq 1.3\text{GeV}$ . However, it depends on the atmospheric neutrino mass scale  $\Delta m_{32}^2$ . The  $\Delta m_{32}^2$  dependence of  $S_{CP}$  is obtained by replacing the axis  $E$  with  $E \times \Delta m_{32}^2 / 10^{-2}\text{eV}^2$  in fig.1. Since the  $CP$  violating effect changes considerably according to values of  $E$ , the new project PS(50GeV) at KEK is very significant for observing  $CP$  violation. Both CERN-Gran Sasso and MINOS experiments are also important because of the non-suppression  $S_{CP}$  as seen in fig.1(b).

## 4 Constraints of Mixings from Present Reactor and Accelerator Data

Analyses of the three flavor neutrino oscillation have been presented recently[18]-[20]. In particular, the quantitative results by Bilenky et al.[19] and Fogli, Lisi and Scioscia [20] are the useful guide for getting constraints of the neutrino mixings. The upper bound of  $J_{CP}^\nu$  is estimated by using these constraints.

Let us begin with discussing constraints from the reactor and accelerator disappearance experiments. Since no indications in favor of neutrino oscillations were found in these experiments, we only get the allowed regions in  $(U_{\alpha i}^2, \Delta m_{31}^2)$  parameter space. Bugey reactor experiment [21] and CDHS [22] and CCFR [23] accelerator experiments give bounds for the neutrino mixing parameters at the fixed value of  $\Delta m_{31}^2$ . We follow the analyses given by Bilenky et al.[19].

Since the  $CP$  violating effect can be neglected in those short baseline experiments as discussed in section 3, we use the following formula without  $CP$  violation for the probability in the disappearance experiments:

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - 4|U_{\alpha i}|^2(1 - |U_{\alpha i}|^2) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right), \quad (5)$$

where  $i=1$  or  $3$  corresponds to hierarchy I or II. The mixing parameters can be expressed

in terms of the oscillation probabilities as [19]

$$|U_{\alpha i}|^2 = \frac{1}{2}(1 \pm \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) , \quad (6)$$

with

$$B_{\nu_\alpha \nu_\alpha} = \{1 - P(\nu_\alpha \rightarrow \nu_\alpha)\} \sin^{-2}\left(\frac{\Delta m_{31}^2 L}{4E}\right) , \quad (7)$$

where  $\alpha = e$  or  $\mu$  and  $i = 1$  or  $3$ . Therefore the parameters  $U_{\alpha i}^2$  at the fixed value of  $\Delta m_{31}^2$  should satisfy one of the following inequalities:

$$|U_{\alpha i}|^2 \geq \frac{1}{2}(1 + \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) \equiv a_\alpha^{(+)} , \quad \text{or} \quad |U_{\alpha i}|^2 \leq \frac{1}{2}(1 - \sqrt{1 - B_{\nu_\alpha \nu_\alpha}}) \equiv a_\alpha^{(-)} . \quad (8)$$

The negative results of Bugey [21], CDHS [22] and CCFR[23] experiments have given the values of  $a_e^{(\pm)}$  and  $a_\mu^{(\pm)}$ , which were presented in ref. [19] and [24].

It is noticed from eq.(8) there are three allowed regions of  $|U_{ei}|^2$  and  $|U_{\mu i}|^2$  as follows:

$$\begin{aligned} (A) \quad & |U_{ei}|^2 \geq a_e^{(+)} , \quad |U_{\mu i}|^2 \leq a_\mu^{(-)} , \\ (B) \quad & |U_{ei}|^2 \leq a_e^{(-)} , \quad |U_{\mu i}|^2 \leq a_\mu^{(-)} , \\ (C) \quad & |U_{ei}|^2 \leq a_e^{(-)} , \quad |U_{\mu i}|^2 \geq a_\mu^{(+)} , \end{aligned} \quad (9)$$

where  $i = 1$ (hierarchy I) or  $3$ (hierarchy II). In addition to these constraints, we should take account of the constraints by E531 [25] and E776 [26] experimental data. These constraints often become severer than the ones of the disappearance experiments as discussed in the next section.

It may be important to comment on the case (A) with hierarchy I. In this case, one has  $U_{e3} \simeq 1$  and then the survival probability of the solar neutrinos is too large to be consistent with the data of GALLEX and SAGE, which have shown less neutrino deficit than the Homestake and Kamiokande experiments [9]. Therefore, this case is an unrealistic one for the neutrino mixings although we include this case in our analyses.

## 5 Upper Bound of $J_{CP}^\nu$

The  $CP$  violating measure  $J_{CP}^\nu$  defined in eq.(4) is also expressed as

$$J_{CP}^\nu = |U_{e1}||U_{e2}||U_{e3}||U_{\mu3}||U_{\tau3}|(|U_{\mu3}|^2 + |U_{\tau3}|^2)^{-1} \sin \phi . \quad (10)$$

This formula is rather suitable for hierarchy II since the experimental constraints are directly given for  $|U_{e3}|$  and  $|U_{\mu3}|$  as seen in eq.(9). For hierarchy I, the constraints for  $|U_{e3}|$  and  $|U_{\mu3}|$  are indirectly given by using unitarity of the mixing matrix. We discuss the upper bound of  $J_{CP}^\nu$  in six cases: cases (A), (B) and (C) with hierarchy I or II. At first, we study the cases with hierarchy II since those are easier for us to estimate  $J_{CP}^\nu$  than the cases with hierarchy I.

The mixing matrix with hierarchy II is written for case (A) as

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & \epsilon_2 & 1 \\ U_{\mu1} & U_{\mu2} & \epsilon_3 \\ U_{\tau1} & U_{\tau2} & \epsilon_4 \end{pmatrix} , \quad (11)$$

where  $\epsilon_i (i = 1 \sim 4)$  are tiny numbers. Then  $J_{CP}^\nu$  is given by

$$J_{CP}^\nu = \epsilon_1 \epsilon_2 \epsilon_3 \epsilon_4 (\epsilon_3^2 + \epsilon_4^2)^{-1} \sin \phi \leq \frac{1}{2} \epsilon_1 \epsilon_2 , \quad (12)$$

where the sign of equality is obtained at  $\epsilon_3 = \epsilon_4$  with  $\sin \phi = 1$ . Here  $\epsilon_3$  is bounded by E776  $\nu_\mu \rightarrow \nu_e$  experiment [26] and  $\epsilon_4$  is given by unitarity. The product  $\epsilon_1 \epsilon_2$  is bounded by unitarity such as

$$\epsilon_1 \epsilon_2 \leq \frac{1}{2} (\epsilon_1^2 + \epsilon_2^2) = \frac{1}{2} (1 - |U_{e3}|^2) \leq \frac{1}{2} (1 - a_e^{(+)}) . \quad (13)$$

Thus the upper bound of  $J_{CP}^\nu$  is given only by  $a_e^{(+)}$ . In this case the atmospheric neutrino anomaly could be attributed to the  $\nu_\mu \rightarrow \nu_\tau$  oscillation if  $|U_{\mu1}| = |U_{\mu2}| = |U_{\tau1}| = |U_{\tau2}| \simeq 1/\sqrt{2}$ . But, it is emphasized that the estimated upper bound of  $J_{CP}^\nu$  is independent of this condition.



For the case (B) with hierarchy II the mixing matrix is given as

$$\mathbf{U} \simeq \begin{pmatrix} U_{e1} & U_{e2} & \epsilon_1 \\ U_{\mu 1} & U_{\mu 2} & \epsilon_2 \\ \epsilon_3 & \epsilon_4 & 1 \end{pmatrix}. \quad (14)$$

We get the bound of  $J_{CP}^\nu$  as follows:

$$J_{CP}^\nu = |U_{e1}| |U_{e2}| \epsilon_1 \epsilon_2 \sin \phi \leq \frac{1}{2} \epsilon_1 \epsilon_2, \quad (15)$$

where the sign of equality is obtained at  $|U_{e1}| = |U_{e2}| = 1/\sqrt{2}$  with  $\sin \phi = 1$ . Then the atmospheric neutrino anomaly could be solved by the large  $\nu_\mu \rightarrow \nu_e$  oscillation. The bound of  $\epsilon_1$  is given by  $a_e^{(-)}$  in eq.(9). On the other hand,  $\epsilon_2$  is bounded by E531  $\nu_\mu \rightarrow \nu_\tau$  experiment [25] since the relevant transition probabilities in the short baseline experiments are given for hierarchy II:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &\simeq 4|U_{e3}|^2 |U_{\mu 3}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right), \\ P(\nu_\mu \rightarrow \nu_\tau) &\simeq 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right). \end{aligned} \quad (16)$$

It may be useful to comment on the possibility of the atmospheric neutrino anomaly by the large  $\nu_\mu \rightarrow \nu_e$  oscillation. The reactor experiments at Bugey [21] and Krasnoyarsk [27] have already excluded some large  $\nu_\mu - \nu_e$  mixing region. The allowed one is  $\sin^2 2\theta_{e\mu} \leq 0.7$  in the case of  $\Delta m_{21}^2 = 10^{-2} \text{eV}^2$ . On the other hands, the data of the atmospheric neutrino anomaly in Kamiokande [7] suggests  $\Delta m_{21}^2 = 7 \times 10^{-3} \sim 8 \times 10^{-2} \text{eV}^2$  and  $\sin^2 2\theta_{e\mu} = 0.6 \sim 1$  for the  $\nu_\mu \rightarrow \nu_e$  oscillation. The overlap region is rather small such as  $\sin^2 2\theta_{e\mu} = 0.6 \sim 0.7$ . Since the first long baseline reactor experiment CHOOZ [28] will soon give the severer constraint for the  $\nu_\mu - \nu_e$  mixing, one can check the possibility of the atmospheric neutrino anomaly due to the large  $\nu_\mu \rightarrow \nu_e$  oscillation.

In the case (C) with hierarchy II the mixing matrix is

$$\mathbf{U} \simeq \begin{pmatrix} U_{e1} & U_{e2} & \epsilon_1 \\ \epsilon_2 & \epsilon_3 & 1 \\ U_{\tau 1} & U_{\tau 2} & \epsilon_4 \end{pmatrix}. \quad (17)$$

Then we have

$$J_{CP}^\nu = |U_{e1}| |U_{e2}| \epsilon_1 \epsilon_4 \sin \phi \leq \frac{1}{2} \epsilon_1 \epsilon_4, \quad (18)$$

where the sign of equality is obtained at  $|U_{e1}| = |U_{e2}| = 1/\sqrt{2}$  with  $\sin \phi = 1$ . In this case the atmospheric neutrino anomaly cannot be solved by the large  $\nu_\mu$  oscillation because both  $\epsilon_2$  and  $\epsilon_3$  are very small. Here  $\epsilon_1$  is bounded by E776  $\nu_\mu \rightarrow \nu_e$  experiment [26] while  $\epsilon_4$  is by E531  $\nu_\mu \rightarrow \nu_\tau$  experiment [25] as seen in eq.(16).

Let us study the cases with hierarchy I, in which the relevant transition probabilities in the short baseline experiments are given instead of eq.(16) as follows:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &\simeq 4|U_{e1}|^2 |U_{\mu 1}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right), \\ P(\nu_\mu \rightarrow \nu_\tau) &\simeq 4|U_{\mu 1}|^2 |U_{\tau 1}|^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right). \end{aligned} \quad (19)$$

For the case (A) with hierarchy I the mixing matrix is

$$\mathbf{U} \simeq \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 \\ \epsilon_3 & U_{\mu 2} & U_{\mu 3} \\ \epsilon_4 & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \quad (20)$$

Then  $J_{CP}^\nu$  is given by

$$J_{CP}^\nu = \epsilon_1 \epsilon_2 |U_{\mu 3}| |U_{\tau 3}| (|U_{\mu 3}|^2 + |U_{\tau 3}|^2)^{-1} \sin \phi \leq \frac{1}{2} \epsilon_1 \epsilon_2, \quad (21)$$

where the sign of equality is obtained at  $|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2}$  with  $\sin \phi = 1$ . The product  $\epsilon_1 \epsilon_2$  is bounded by unitarity such as

$$\epsilon_1 \epsilon_2 \leq \frac{1}{2}(\epsilon_1^2 + \epsilon_2^2) = \frac{1}{2}(1 - |U_{e1}|^2) \leq \frac{1}{2}(1 - a_e^{(+)}). \quad (22)$$

Thus we get the same upper bound of  $J_{CP}^\nu$  as the one in the case (A) with hierarchy II. The atmospheric neutrino anomaly could be attributed to the  $\nu_\mu \rightarrow \nu_\tau$  oscillation if  $|U_{\mu 2}| = |U_{\mu 3}| = |U_{\tau 2}| = |U_{\tau 3}| \simeq 1/\sqrt{2}$ .

For the case (B) with hierarchy I the mixing matrix is written as

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & U_{e2} & U_{e3} \\ \epsilon_2 & U_{\mu 2} & U_{\mu 3} \\ 1 & \epsilon_3 & \epsilon_4 \end{pmatrix}. \quad (23)$$

Then we get

$$J_{CP}^\nu = \epsilon_1 \epsilon_4 |U_{e2}| |U_{e3}| |U_{\mu 3}| (|U_{\mu 3}|^2 + \epsilon_4^2)^{-1} \sin \phi \leq \frac{1}{\sqrt{2}} \epsilon_1 \epsilon_4, \quad (24)$$

where the sign of equality is obtained at  $|U_{e2}| = |U_{e3}| = |U_{\mu 3}| = 1/\sqrt{2}$  with  $\sin \phi = 1$ . However, this bound is not exact one in contrast to previous cases. We checked numerically that eq.(24) gives roughly the maximum value by using the present bound of  $\epsilon_4$ . The magnitude of  $\epsilon_1$  is bounded by  $a_e^{(-)}$  in eq.(9) while  $\epsilon_4$  is bounded by unitarity such as

$$\epsilon_4^2 = \epsilon_1^2 + \epsilon_2^2 - \epsilon_3^2 \leq \epsilon_1^2 + \epsilon_2^2, \quad (25)$$

where  $\epsilon_2$  is bounded by E531  $\nu_\mu \rightarrow \nu_\tau$  experiment [25]. The upper bound of  $J_{CP}^\nu$  is different from the one in the case (B) with hierarchy II. The atmospheric neutrino anomaly could be solved by the large  $\nu_\mu \rightarrow \nu_e$  oscillation.

For the case (C) with hierarchy I the mixing matrix is

$$\mathbf{U} \simeq \begin{pmatrix} \epsilon_1 & U_{e2} & U_{e3} \\ 1 & \epsilon_2 & \epsilon_3 \\ \epsilon_4 & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \quad (26)$$

Then  $J_{CP}^\nu$  is given by

$$J_{CP}^\nu = \epsilon_1 \epsilon_3 |U_{e2}| |U_{e3}| |U_{\tau 3}| (\epsilon_3^2 + |U_{\tau 3}|^2)^{-1} \sin \phi \leq \frac{1}{\sqrt{2}} \epsilon_1 \epsilon_3, \quad (27)$$

where the sign of equality is obtained at  $|U_{e2}| = |U_{e3}| = |U_{\tau3}| \simeq 1/\sqrt{2}$  with  $\sin \phi = 1$ . This bound is also not exact one as well as the case (B) although it is roughly the maximum value. Here  $\epsilon_1$  is bounded by E776  $\nu_\mu \rightarrow \nu_e$  experiment [26]. On the other hand,  $\epsilon_3$  is bounded by unitarity

$$\epsilon_3^2 = |U_{\mu3}|^2 = 1 - |U_{\mu1}|^2 - |U_{\mu2}|^2 \leq 1 - a_\mu^{(+)} , \quad (28)$$

where  $a_\mu^{(+)}$  is given by the disappearance experiments as seen in eq.(9). The upper bound of  $J_{CP}^\nu$  is different from the one in the case (C) with hierarchy II.

Thus we obtain the upper bounds of  $J_{CP}^\nu$  for six cases which are allowed by the present short baseline experiments.

## 6 Numerical Results of $CP$ Violation

Now we can calculate the upper bound of  $\Delta P \equiv P(\nu_\mu \rightarrow \nu_\tau) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$ , which is the direct measurement of  $CP$  violation. Since the upper bounds of  $J_{CP}^\nu$  have been given for fixed  $\Delta m_{31}^2$ , the upper bounds of  $\Delta P$  are also presented for  $\Delta m_{31}^2$  with fixing  $L$ ,  $E$  and  $\Delta m_{32}^2$  ( for hierarchy I) or  $\Delta m_{21}^2$  ( for hierarchy II). In fig.2(a), we show numerical results for cases (A), (B) and (C) of hierarchy I with  $L = 250\text{Km}$  and  $\Delta m_{32}^2 = 10^{-2}\text{eV}^2$ . Fig.2(b) corresponds to hierarchy II. Here we used the energy band of  $E = 1 \sim 1.5\text{GeV}$  in the energy spectrum of the incident neutrino, which is expected in KEK-PS [4]. Then we get the averaged value  $S_{CP} = 0.725$  for  $\Delta m_{31}^2 = 1 \sim 25\text{eV}^2$ , which is used in our calculation to avoid long CPU time due to the oscillatory integrand. Therefore, one should take into consider 5% error in the results of Figs. (a) and (b).

**Fig. 2(a) and (b)**

The weakest bound is given in the case (B) with hierarchy I, in which the bound is almost determined only by Bugey reactor disappearance experiment [21]. In this case  $\Delta P$  could be  $10^{-1}$ , which can be observed in KEK-SuperKamiokande experiment. Then the atmospheric neutrino anomaly is due to the large  $\nu_\mu \rightarrow \nu_e$  oscillation. The first long baseline reactor experiment CHOOZ [28] will soon test this possibility by presenting the severer constraint of the  $\nu_\mu - \nu_e$  mixing.

The observation of the  $CP$  violating effect is not expected in the case (C) for both hierarchies I and II since the upper bounds are around or below  $10^{-2}$ . In addition, the atmospheric neutrino anomaly could not be explained by the large neutrino mixing.

If the large  $\nu_\mu \rightarrow \nu_\tau$  oscillation causes the atmospheric neutrino anomaly, the case (A) is preferred. The upper bound is around 0.03, which is same for both hierarchies. Since this bound is determined only by the reactor disappearance experiments [21], it will be improved by new disappearance experiments.

It is remarked that the estimated upper bounds of  $J_{CP}^\nu$  are given by the maximal mixing such as  $|U_{\alpha i}| \simeq 1/\sqrt{2}$  except for the case (A) with hierarchy II. If the atmospheric neutrino anomaly is not due to the large neutrino mixing, the  $CP$  violating effect is reduced. The situation is different in the case (A) with hierarchy II. In this case, the upper bound has been obtained without assuming the large neutrino mixing.

In our analyses, we do not take account of the new experimental data given by LSND [14]. Even if the data is included, our obtained bounds do not almost change.

## 7 Matter Effect

The general discussion of the matter effect in the long baseline experiments was given by Kuo and Pantaleone [29]. The data in those experiments include the background

matter effect which is not  $CP$  invariant. Therefore, it is very important to investigate the matter effect in order to estimate the  $CP$  violation effect originated by the phase of the neutrino mixing matrix. The matter effect of the earth should be carefully analyzed since the effect considerably depends on the mass hierarchy and mixings as well as the incident energy of the neutrino.

We estimate the matter effect on the transition probabilities by switching off  $CP$  violation due to the mixing matrix. Then,  $\Delta P$  is given by the only matter effect. If the estimated  $\Delta P$  is comparable to the ones in the previous section, one should consider how to extract the matter effect from the data.

The matter effect in the long baseline accelerator experiments is rather easily estimated by assuming the constant electron density. The effective mass squared in matter  $M_m^2$  for neutrino energy  $E$  in weak basis [29] is

$$\mathbf{M}_m^2 = U_m \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U_m^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (29)$$

where  $A = 2\sqrt{2}G_F n_e E$ . We use the constant electron density  $n_e = 2.4 \text{ mol/cm}^3$ . For antineutrinos, the effective mass squared is given by replacing  $A \rightarrow -A$  and  $U_m \rightarrow U_m^*$ . The effective mixing matrix without the  $CP$  violating phase  $U_m$  is written by

$$U_m = \begin{pmatrix} c_{m13}c_{m12} & c_{m13}s_{m12} & s_{m13} \\ -c_{23}s_{m12} - s_{23}s_{m13}c_{m12} & c_{23}c_{m12} - s_{23}s_{m13}s_{m12} & s_{23}c_{m13} \\ s_{23}s_{m12} - c_{23}s_{m13}c_{m12} & -s_{23}c_{m12} - c_{23}s_{m13}s_{m12} & c_{23}c_{m13} \end{pmatrix}, \quad (30)$$

where  $s_{mij} \equiv \sin \theta_{ij}^m$ ,  $c_{mij} \equiv \cos \theta_{ij}^m$  for effective mixings in the matter and  $s_{ij} \equiv \sin \theta_{ij}$ ,  $c_{ij} \equiv \cos \theta_{ij}$  for vacuum mixings. For example, the effective mixing angle  $s_{m12}$  is given in terms of vacuum mixings as

$$\sin 2\theta_{12}^m = \frac{\Delta m_{21}^2 \sin 2\theta_{12}}{\sqrt{(A \cos^2 \theta_{13} - \Delta m_{21}^2 \cos 2\theta_{12})^2 + \Delta m_{21}^2 \sin^2 2\theta_{12}}} \quad (31)$$

to zeroth order in  $A \sin 2\theta_{13}$ .

In the case of hierarchy I ( $m_3 \simeq m_2 \geq 1\text{eV}$ ), the matter effect is expected to be small because  $m_{21}^2 \gg A \simeq 5 \times 10^{-4}\text{eV}^2$ . In fact, the obtained  $\Delta P$  is at most  $5 \times 10^{-3}$  for three cases of (A), (B) and (C). Therefore, the matter effect do not disturb the information of  $CP$  violation originated by the  $CP$  violating phase in the neutrino mixing matrix if the observed  $\Delta P$  is not far from our estimated upper bounds in section 6 (see Fig.2).

However, the case with hierarchy I is another. Since the value of  $A$  is not negligible compared to  $m_{21}^2 \simeq 10^{-2}\text{eV}^2$ , the matter effect is expected to be important. We show the matter effect  $\Delta P$  versus  $s_{12}$  for the typical parameters in Figs.3(a) and (b). Here the solid curves denote the matter effect  $\Delta P(\nu_\mu \rightarrow \nu_\tau)$  and the dashed curves denote  $\Delta P(\nu_\mu \rightarrow \nu_e)$ , where  $\Delta P(\nu_\alpha \rightarrow \nu_\beta) \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ <sup>3</sup>. The parameters are fixed to be  $m_{31}^2 = 2.25\text{eV}^2$ ,  $m_{21}^2 = 10^{-2}\text{eV}^2$ ,  $L = 250\text{Km}$  and  $E = 1.2\text{GeV}$ . The vacuum mixing angles are taken to be  $s_{13} = 0.96$  and  $s_{23} = 1/\sqrt{2}$  for case (A) and  $s_{13} = 0.15$  and  $s_{23} = 0.12$  for case (B).

**Fig. 3(a) and (b)**

In case (A), the  $\nu_\mu - \nu_\tau$  mixing is maximum at  $s_{12} = 0$  or 1. Then, the matter effect  $\Delta P(\nu_\mu \rightarrow \nu_\tau)$  increases up to  $7.5 \times 10^{-3}$ . For the  $\nu_\mu - \nu_e$  mixing which is very small in this case, the matter effect  $\Delta P(\nu_\mu \rightarrow \nu_e)$  is tiny. If the  $CP$  violation effect  $\Delta P$  is larger than  $10^{-2}$  as shown in Fig. 2(b), the matter effect does not dominate  $\Delta P$ . The smallness of the matter effect is due to the suppressed  $Ac_{13}^2$  in eq.(31) ( $c_{13} \ll 1$  in case (A)).

In case (B), the  $\nu_\mu - \nu_e$  mixing is maximum at  $s_{12} = 1/\sqrt{2}$ . The matter effect  $\Delta P(\nu_\mu \rightarrow \nu_e)$  could be  $8 \times 10^{-2}$ . Since the  $\nu_\mu - \nu_\tau$  mixing is very small in this case, the

---

<sup>3</sup> The  $CP$  non-invariant quantity  $|\Delta P(\nu_\mu \rightarrow \nu_\tau)|$  is different from  $|\Delta P(\nu_\mu \rightarrow \nu_e)|$  for the matter effect. On the other hand, those have same magnitudes for  $CP$  violation originated by the phase in the neutrino mixing matrix as seen in Eq.(2).

matter effect  $\Delta P(\nu_\mu \rightarrow \nu_\tau)$  is also small. Since the matter effect is very large compared to the results in Fig.2(b), it is difficult to get the information of the  $CP$  violating phase in the mixing matrix.

In case (C), both  $\nu_\mu - \nu_e$  and  $\nu_\mu - \nu_\tau$  mixings are very small, and so the matter effects  $\Delta P$ 's are also small, at most  $5 \times 10^{-4}$ . In this case, there is no hope to observe the neutrino oscillation in the planned long baseline accelerator experiments.

Thus, the matter effect becomes important to observe  $CP$  violation in the case of hierarchy II. However, recent works of the pattern of the neutrino masses and mixings [10][15] may exclude hierarchy II. If hierarchy I is realized for the neutrino masses, the matter effect does not almost modify our results in Fig.2(a).

## 8 Conclusions

We have studied the direct measurements of  $CP$  violation originated by the phase of the neutrino mixing matrix in the long baseline neutrino oscillations. In those experiments, the  $CP$  violating effect is not suppressed if the highest neutrino mass scale is taken to be  $1 \sim 5\text{eV}$ , which is appropriate for the cosmological HDM. The upper bounds have been calculated for three cases (A), (B), (C) in hierarchies I and II, where mixings are constrained by the recent short baseline ones. The estimated upper bounds are larger than  $10^{-2}$ , which is observable in the long baseline accelerator experiments. The new reactor disappearance experiments will provide severer bound in the near future.

The matter effect on  $CP$  violation is also calculated. The effect is not significant for hierarchy I, but for hierarchy II. The recent works of the neutrino masses and mixings suggest the case (A) with hierarchy I. In this case, the matter effect on  $CP$  violation is negligible if the observed  $\Delta P$  is close to our estimated upper bound.



## Acknowledgments

I would like to thank J. Pantaleon for his critical comments on the matter effect. I thank A. Smirnov and H. Nunokawa for the quantitative discussion of the matter effect. I also thank S.M. Bilenky and W. Grimus for discussing the  $CP$  violating experiments. I wish to acknowledge the hospitality of DESY theory group, in particular, A. Ali. This research is supported by Alexander von Humboldt foundation (Germany) and the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan(No. 07640413).

## References

- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**(1973)652.
- [2] S. Pakvasa, in *Proc of the XX International Conference of High Energy Physics*, Madison, Wisconsin, USA, 1980, **Part 2** (1980)1165, edited by L. Durand and L.G. Pondrom, AIP, New York;  
V. Barger, K. Whisnant and R.J.N. Phillips, Phys. Rev. Lett. **45**(1980)2084;  
Phys. Rev. **D23**(1981)2773;  
S.M. Bilenky and F. Nidermaler, Sov. J. Nucl. Phys. **34** (1981)606.
- [3] S. Parke, Fermilab-Conf-93/056-T(hep-ph/9304271)(1993);  
L. Camilleri, CERN preprint, CERN-PPE/94-87(1994);  
ICARUS Collaboration, Gran Sasso Lab. preprint LNGS-94/99-I(1994);  
S. Wojcicki(MINOS), invited talk at XVII Conference on Neutrino Physics and Neutrino Astrophysics, June 13-19, 1996, Helsinki.
- [4] Y. Suzuki(KEK), invited talk at XVII Conference on Neutrino Physics and Neutrino Astrophysics, June 13-19, 1996, Helsinki.
- [5] K.S. Hirata et al., Phys. Lett. **205B**(1988)416; **280B**(1992)146;  
D. Casper et al., Phys. Rev. Lett. **66**(1991)2561;  
R. Becker-Szendy et al., Phys. Rev. **D46**(1992)3720.
- [6] NUSEX Collaboration, Europhys. Lett. **8**(1989)611; ibidem **15**(1991)559;  
SOUDAN2 Collaboration, Nucl. Phys. **B35**(Proc. Suppl.)(1994)427;  
ibidem **38**(1995)337;  
Fréjus Collaboration, Z. Phys. **C66**(1995)417;  
MACRO Collaboration, Phys. Lett. **357B**(1995)481.

- [7] Y. Fukuda et al., Phys. Lett. **335B**(1994)237.
- [8] R. N. Mohapatra and G. Senjanovic, Z. Phys. **C17**(1983) 53;  
 R. Holman, G. Lazarides and Q. Shafi, Phys. Rev. **D27**(1983) 995;  
 Q. Shafi and F. W. Stecker, Phys. Rev. Lett. **53**(1984) 1292;  
 R. Schaefer, Q. Shafi and F.W. Stecker, Astrophys.J.**347**(1989)575;  
 J.A. Holtzman and J.R. Primack, Astrophys.J.**405**(1993)428;  
 A. Klypin, J. Holtzman, J. Primack and E. Regos, Astrophys.J.**416**(1993)1;  
 Y.P. Jing, H.J. Mo, G. Borner and L.Z. Fang, Astron. Astrophys.**284** (1994)703;  
 J.R. Primack, J. Holtzman, A. Klypin and D. O. Caldwell, Phys. Rev. Lett.  
**74**(1995)2160;  
 K.S. Babu, R.K. Schaefer and Q. Shafi, Phys. Rev. **D53**(1996) 606;  
 C.Y. Cardall and G.M. Fuller, Phys. Rev. **D53**(1996) 4421.
- [9] GALLEX Collaboration, Phys. Lett. **327B**(1994)377;  
 SAGE Collaboration, Phys. Lett. **328B**(1994)234;  
 Homestake Collaboration, Nucl. Phys. **B38**(Proc. Suppl.)(1995)47;  
 Kamiokande Collaboration, Nucl. Phys. **B38**(Proc. Suppl.)(1995)55.
- [10] J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. **B406**(1993)409;  
 D.O. Caldwell and R.N. Mohapatra, Phys.Rev. **D48** (1993)3259,  
 ibidem **D50**(1994)3477;  
 J.J. Gomez-Cadenas and M.C. Gonzalez-Garcia, preprint hep-ph/9504246(1995);  
 S. Goswami, preprint hep-ph/9507212(1995);  
 N. Okada and O. Yasuda, TMUP-HEL-9605, hep-ph/9606411(1996);  
 N. Okada, TMUP-HEL-9607, hep-ph/9606221(1996);  
 S.M. Bilenky, C. Giunti and W. Grimus, UWThPh-1996-42, hep-ph/9607372(1996).

- [11] J. Arafune and J. Sato, preprint ICRR-369-96-20, hep-ph/9607437(1996).
- [12] C. Jarlskog Phys. Rev. Lett. **55**(1985)1839.
- [13] Particle Data Group, Phys. Rev. **D50**(1994)1173.
- [14] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. **75**(1995)2650;  
preprint, LA-UR-96-1326(1996)(nucl-ex/9605001);  
J. E. Hill, Phys. Rev. Lett. **75**(1995)2654.
- [15] S.T. Petcov and A.Yu. Smirnov, Phys. Lett. **322B**(1994)109;  
D.O. Caldwell and R.N. Mohapatra, Phys. Lett. **354B**(1995)371;  
G. Raffelt and J. Silk, Phys. Lett, **366B**(1996)429.
- [16] KARMEN Collaboration, Nucl. Phys. **B38**(Proc. Suppl.)(1995)235.
- [17] K. Winter, Nucl. Phys. **B38**(Proc. Suppl.)(1995)211;  
M. Baldo-Ceolin, ibidem **35**(1994)450;  
L. DiLella, Nucl. Phys. **B31**(Proc. Suppl.)(1993)319.
- [18] H. Minakata, Phys. Lett. **356B**(1995)61; Phys. Rev. **D52**(1995)6630.
- [19] S.M. Bilenky, A. Bottino, C. Giunti and C. W. Kim, Phys. Lett. **356B**  
(1995)273; DFTT 2/96(JHU-TIPAC 96002)(1996).
- [20] G.L. Fogli, E. Lisi and G. Scioscia, Phys. Rev. **D52**(1995)5334.
- [21] B. Achkar et al., Nucl. Phys. **B434**(1995)503.
- [22] CDHS Collaboration, F. Dydak et al., Phys. Lett. **134B**(1984)281.
- [23] CCFR Collaboration, I.E. Stockdale et al., Phys. Rev. Lett. **52**(1984)1384;  
Z. Phys. **C27**(1985)53.

- [24] M. Tanimoto, Phys. Rev. **D53** (1996)6632.
- [25] E531 Collaboration, N. Ushida et al., Phys. Rev. Lett. **57**(1986)2897.
- [26] E776 Collaboration, L. Borodovsky et al., Phys. Rev. Lett. **68**(1992)274.
- [27] G.S. Vidyakin et al., Pis'ma Zh.Eksp. Thor. Fiz. **59**(1994)364;  
JETP Lett. **59**(1994)390.
- [28] R.I. Steinberg, Proc. of the 5-th Int.Workshop on neutrino Telescopes, Venice,  
Italy, ed. by M. Baldo-Ceolin, INFN, Padua(1993)209.
- [29] T.K. Kuo and J. Pantaleo, Phys. Lett. **198B**(1987)406.

## Figure Captions

### Figure 1:

Dependences of  $S_{CP}$  on the neutrino energy  $E$  for (a)  $L = 250\text{Km}$  and (b)  $L = 730\text{Km}$  with  $\Delta m_{31}^2 = 2.25\text{eV}^2$  and  $\Delta m_{32}^2 = 10^{-2}\text{eV}^2$ , which are shown by solid curves. The dashed curves denote  $\sin^2(\Delta m_{32}^2 L/4E)$ . Those are averaged over energy spread of 20% of the neutrino energy.

### Figure 2:

Upper bounds of  $\Delta P$  versus  $\Delta m_{31}^2$  for (a) hierarchy I and (b) hierarchy II. The solid, dashed and dashed-dotted curves denote the cases (A), (B) and (C), respectively. We take  $E = 1 \sim 1.5\text{GeV}$  and  $L = 250\text{Km}$  with  $\Delta m_{32}^2 = 10^{-2}\text{eV}^2$  for hierarchy I and  $\Delta m_{21}^2 = 10^{-2}\text{eV}^2$  for hierarchy II.

### Figure 3:

The matter effect  $\Delta P$  versus  $s_{12}$  in hierarchy II for (a) case (A) and (b) case (B). The solid and dashed curves denote  $\Delta P(\nu_\mu \rightarrow \nu_\tau)$  and  $\Delta P(\nu_\mu \rightarrow \nu_e)$ , respectively. Here  $\Delta m_{31}^2 = 2.25\text{eV}^2$ ,  $\Delta m_{21}^2 = 10^{-2}\text{eV}^2$ ,  $E = 1.2\text{GeV}$  and  $L = 250\text{Km}$  are taken.

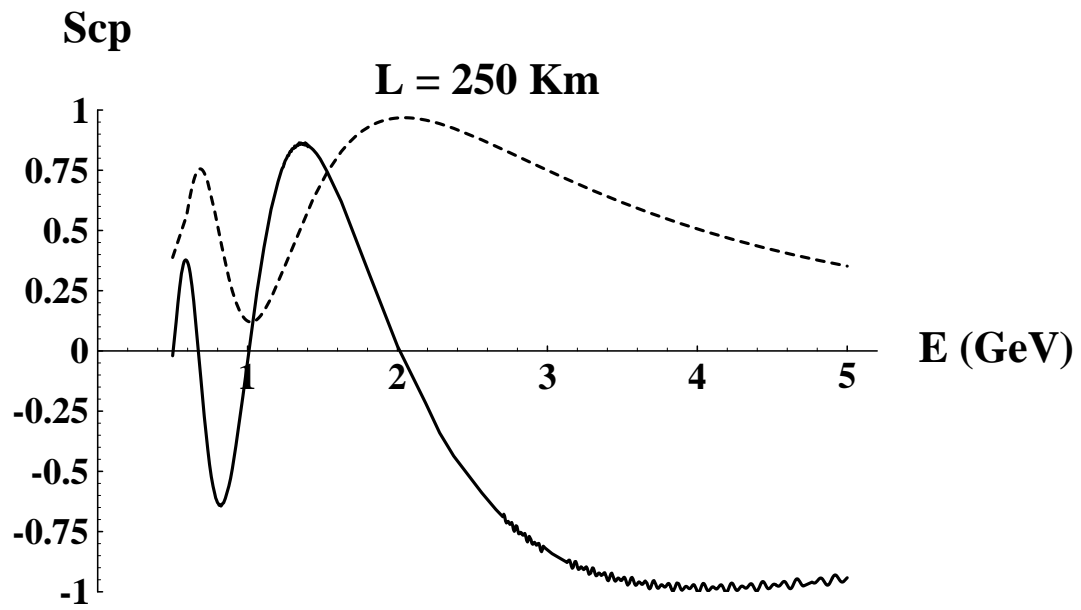


Fig. 1 (a)

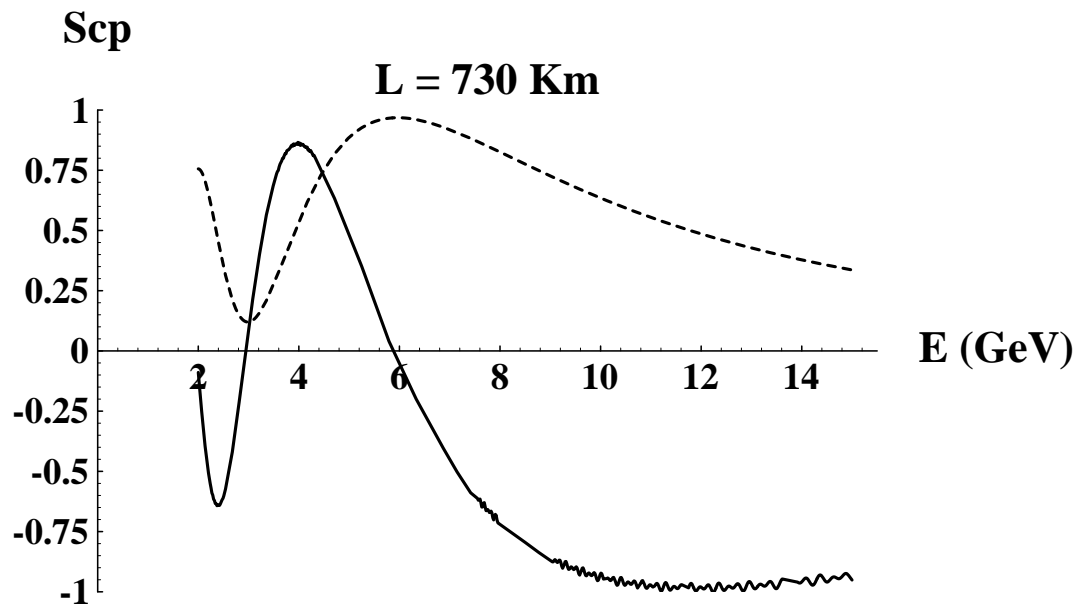


Fig. 1 (b)



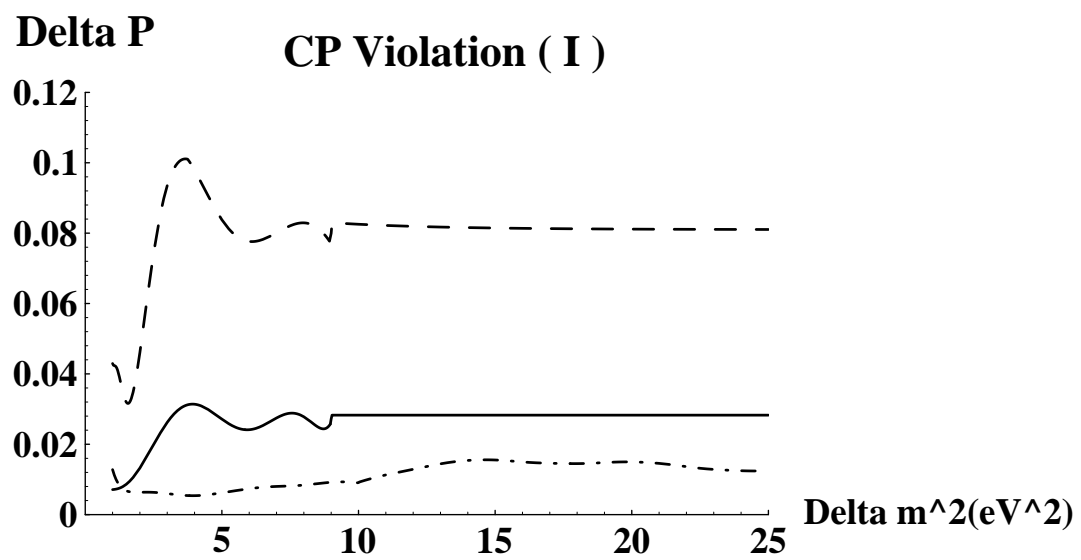


Fig. 2 (a)

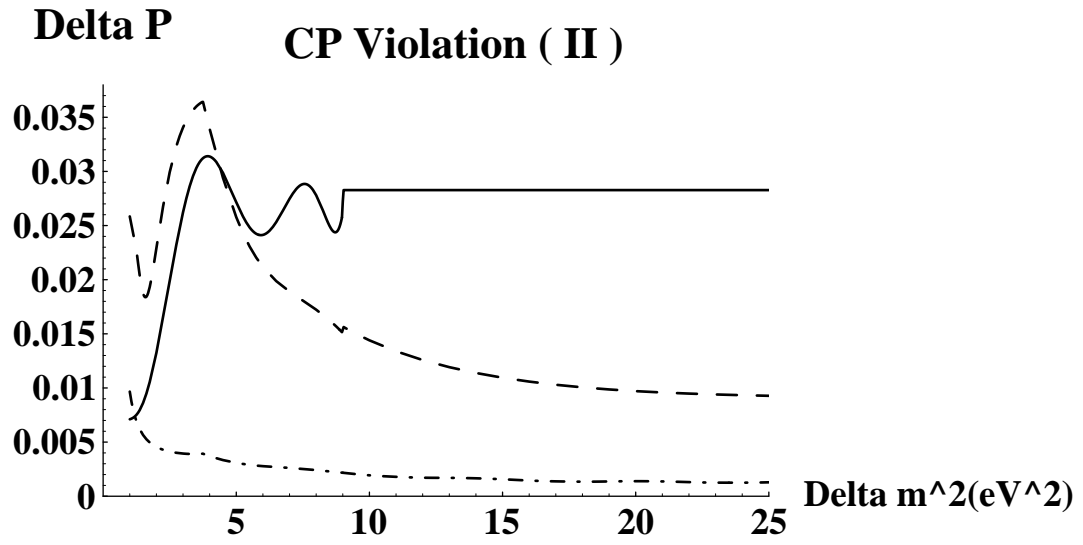


Fig. 2 (b)

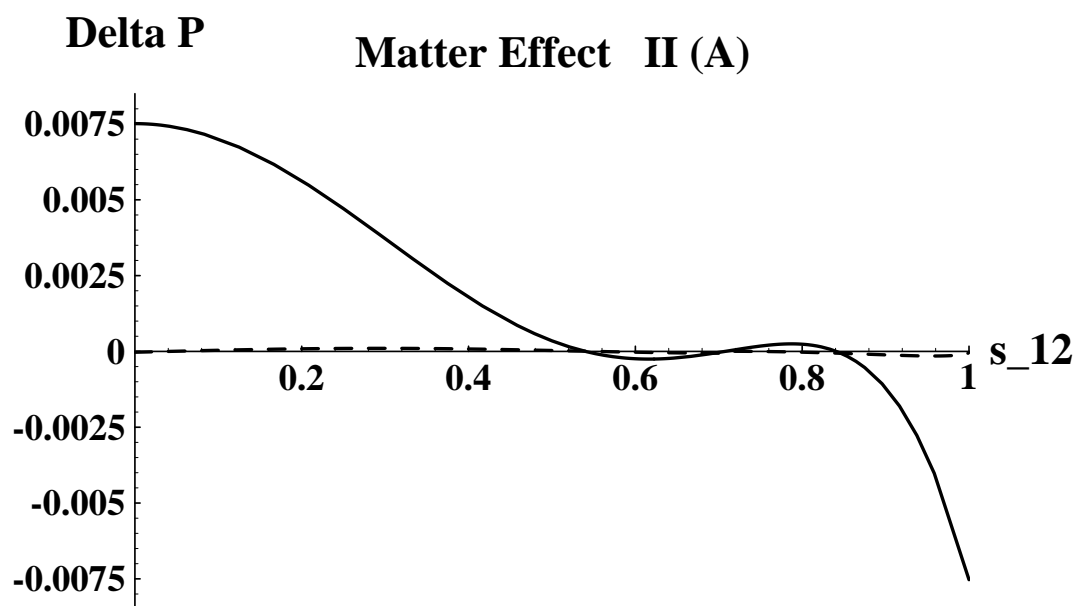


Fig. 3 (a)

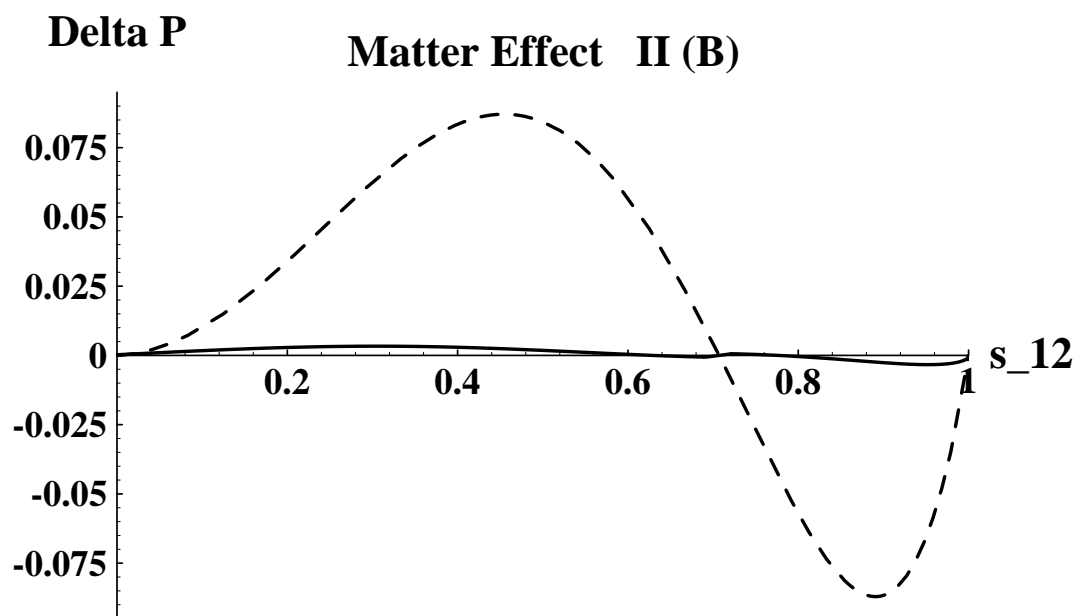


Fig. 3 (b)

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9605413v2>

This figure "fig2-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9605413v2>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9605413v2>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9605413v2>